TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 367

APPLICATION OF THE "MAGNUS EFFECT" TO THE

WIND PROPULSION OF SHIPS

By L. Prandtl

From "Die Naturwissenschaft," Vol. XIII
June 2, 1925

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Washington
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By L. Prandtl.

The Flettner wind-driven rotor ship, which is on every tongue, as a result of the unusually zealous newspaper propaganda, has suddenly aroused much popular interest in the theory of flow. The effect of the Flettner rotor can not be explained in accordance with the prevailing views on wind pressure, although it is claimed to exert as much force as a sail having 10-15 times as large a frontal area.

Though the modern theory of flow can not yet fully explain the mysterious phenomenon, it was nevertheless instrumental in discovering the great forces of the rotating cylinder. Since the knowledge of the laws of flow is not yet sufficiently wide-spread and since such knowledge is necessary to enable us to arrive at any real explanation of the phenomenon, I gladly improve this opportunity to introduce the reader to these laws, as regards the subject under consideration.

This remarkable phenomenon was first observed in connection with a rotating sphere hurled through the air, the deviations of which from the theoretical trajectory had long been

* "Magnuseffekt und Windkraftschiff," a lecture delivered before "Die Göttinger Physikalische Gesellschaft," Nov. 17, 1924, and published in "Die Naturwissenschaft," Vol. XIII, pp. 93-108.

known by artillerists and ball players. When real "balls" were still used, artillerists had early noted certain irregular deviations in their trajectories. B. Robins expressed his opinion in 1742 that these deviations were due to the rotation of the balls. He subsequently demonstrated experimentally the truth of his assumption. Toward 1830, in order to control the formerly very irregular rotations, bullets with an eccentrically located center of gravity were used. It was found that when such a ball was loaded with the center of gravity down, the shot regularly fell short, and that when the center of gravity was up the shot was long, since the pressure of the powder gases (being directed against the center of the ball) caused a downward rotation in the first case and an upward rotation in the second case. In like manner, placing the center of gravity on the right or left caused a corresponding deflection to the right or left. This deflection could not be explained by the assumption of a lateral impulse at the mouth of the gun, because experiments with disks placed at different distances from the muzzle showed that the trajectory was continuously deflected.

In order to settle the question, the well-known Berlin physicist, G. Magnus, a teacher of Helmholtz, performed several laboratory experiments in 1852 (Cf. the article on Gustav Magnus by P. Pringsheim, "Die Naturwissenschaften," Vol. XIII, p. 49, 1925). In one of his experiments, he set a brass cylinder (mounted between pivots, so as to be capable of rapid rotation by means of a cord) on a light rotatable arm and directed an air stream against the cylinder by means of a small centrifugal blower. The cylinder yielded in the direction perpendicular to the air stream and to the axis of the cylinder and always toward the side on which the peripheral motion of the cylinder coincided with the direction of the air stream. The direction of the deflection agreed with the shooting experiments. The valué of the deflecting force, which Magnus did not measure, seemed to him to be of sufficient magnitude to account for the deflections of spherical projectiles.* Since then it has been customary to designate the whole group of phenomena by the term "Magnus effect." The service of Magnus, in first furnishing an experimental demonstration of the phenomenon, thus received appropriate recognition. The knowledge of the effect of rotating balls in ball games antedated even the observations by artillerists. This phenomenon is very striking in tennis, it being common for every skillful player to "cut" the ball so as to make it deviate from its natural path.** If the ball is given a cutting stroke on the right, it is deflected toward the left and vice versa. If cut underneath,

* Magnus also investigated the deflections of rotating oblong projectiles by special experiments and theoretical deductions. In the popular presentations, as published in the newspapers, these were often confused with the Magnus effect, with which they had nothing to do.

**According to G. T. Walker, it was mentioned in 1671 that a "cut" tennis ball describes a curve.

the ball will go farther and, if cut on top, it will go shorter than otherwise. According to G. T. Walker, this phenomenon is even more striking in golf. When a golf ball is cut underneath it goes farther than it could otherwise do with the same initial velocity. The first golf stick (the "driver") is therefore slightly concave on top. In a faulty lateral cut, it is said to be possible for the deflection to amount to as much as 70 meters (230 feet).

The Magnus effect can also be easily demonstrated in the lecture room. The only apparatus required is a cylinder made by gluing together the edges of a sheet of paper. This cylinder is thrown in such a way as to give it a rapid rotary motion. It is preferable to throw it forward, as in bowling, when it will describe a path similar to Fig. 2. The phenomenon is particularly noticeable with a very long cylinder. It is advisable to close the ends with projecting cardboard disks which increase the rigidity and inertia of the cylinder and improve it aerodynamically. If we wind such a cylinder (Fig. 1) in the middle of which a small tongue is cut, with a thread whose ends are attached to an overhead bar, the cylinder, in falling, will then be deflected toward the side. Also a triangular paper prism, likewise provided with end disks, when held between the thumb and finger of one hand and snapped with a finger of the other hand, as indicated by the arrow in Fig. 2, will undergo a very striking deflection. (These experiments were shown in connection with the lecture.)

Artillerists formerly tried to explain this phenomenon by claiming that the deflection was due to the increased friction produced by the "cushion of condensed air" formed in front of the projectile. S. D. Poisson demonstrated, however, that this friction was by no means sufficient to produce such an effect. Moreover, the above-mentioned experiments with eccentric projectiles demonstrated that the deflection was in exactly the opposite direction to that required by the "cushion theory."

Magnus gave, in connection with his experiments, an explanation which makes the effect a little more comprehensible, but which, due to the primitive state of the theory of flow at that time, is no longer satisfactory.* The explanation given by Lord Rayleigh in 1877 in connection with the flight paths of "cut" tennis balls is much more satisfactory. In the meantime the theory of flow had been considerably developed by Helmholz,

* Magnus at first proceeded on the basis that an air stream, which is directed against a resting cylinder perpendicular to its axis, blows laterally-placed candle flames or small streamers toward the cylinder. (Obviously, the air stream must be thin, so that the candles or streamers can be placed outside of it.) Magnus concludes, from the behavior of the flames and streamers, that "the motion of the air past the surface of the cylinder does not, as generally assumed, increase the pressure against the latter, but, on the contrary, reduces it in the direction perpendicular to the air stream and indeed all the more, the greater the velocity of the air." He then continues "If the cylinder is not rotating, the reduction of the pressure is the same on both sides. If it is rotating, however, the velocity and, consequently, the pressure reduction are greater on the side which is moving with the air, than on the side which is moving against the air." He had noticed a damming up of the air on the latter side and therefore assumed the same pressure as when two opposite streams of water meet.

Sir William Thompson and others, though their deductions applied almost exclusively to an "ideal fluid" without viscosity or change in volume. Lord Rayleigh's calculations had to do with the flow of such an ideal fluid past a cylinder infinitely extended in the direction of its axis. He considered the flow conditions, which arise when the ordinary potential flow is combined with a circulation flow, and computed the force exerted on the cylinder from the pressure distribution on the cylinder. Lord Rayleigh himself calls attention to the fact that it is a weak point in his theory that the equations are correct only in the absence of viscosity, but that, according to a proposition of Thompson, the circulatory motion could not occur in the absence of viscosity or, if it were once present, it could not change. The origin of the flow, for which we must obviously hold the viscosity responsible, was consequently left unexplained. Nevertheless, the conclusions of Lord Rayleigh are very instructive and it is useful for us to consider them, as also the related theories on the motion of an ideal fluid, more closely, since the actual fluids, like water and air, have only a very slight viscosity, so that in the most favorable cases, any discovery concerning the motion of an ideal fluid furnishes approximately an accurate conclusion for a real fluid. As to whether we have such a favorable case under consideration still requires special investigation. There is more to be said later on this subject.

The ordinarily tested flows of an ideal fluid, namely, the flows which are produced in an originally quiet fluid, by the motion of bodies in it or by the action of pressure on its surface, are of the same geometrical kind as the currents of electricity in a homogeneous solid conductor (or as the magnetic field in a space with unchangeable permeability). They are deducible from a potential which satisfies the Laplace differential equation. The essential characteristic of the potential flow is that the individual fluid particles do not rotate. This is closely connected with freedom from viscosity, without the aid of which a fluid particle can not be set in rotation.

The two kinds of flow, on which Rayleigh based his calculations, are shown in Figs. 3-4. A flow of the form shown in Fig. 3 can be produced by sending an electric current through an iron sheet with a round hole cut in its center. For this purpose, strips of good-conducting copper must be soldered to the right and left edges of the poor-conducting iron sheet. The flow shown in Fig. 4 can be produced electrically only by slitting the sheet with the round hole, along a radius and soldering the copper strips on the right and left sides of the slit. Magnetic fields of the form shown in Fig. 4 are well known. The magnetic field generated by an electric current passing through a wire perpendicular to the plane of the drawing is of this kind.

The flow, which is the real subject of Rayleigh's investigation, is obtained by superposing the two flows shown in Figs. 3-4. By this is meant the flow whose velocity at every point is the result of the combination of the two component velocities according to the parallelogram of velocities. The potential is produced simply by adding the two potential values at each point in space. The streamline system is produced, when the two streamline systems are so drawn, one over the other, that the quantity of fluid passing per second between every two streamlines is everywhere the same, simply by drawing the diagonal curves, as shown in Fig. 5. As the result of superposing the flows shown in Figs. 3-4, various forms are produced, according to the intensity of the circulatory motion (Fig. 4). One form, with a moderate circulation, is shown in Fig. 6; another, with a stronger circulation, in Fig. 7.

In order to understand what is accomplished with these flows, we must consider the pressure distribution in a flowing ideal fluid. In the first place the pressure at one and the same point is the same in all directions, exactly the same as in a fluid at rest. The pressures at two different points, however, generally differ. We will disregard the weight of the fluid, so that we will here understand by the term "pressure" only the difference between the pressure when in motion and when at rest. This difference may be either positive or negative and we will employ the terms "positive pressure" and "negative pressure" in this connection. When the pressure continuously falls from any point A to any point B on the same streamline, each fluid particle has a somewhat greater pressure behind it than in front of it and is therefore accelerated in the direction of the decreasing pressure. If it already has, at A, a velocity toward B, this velocity will continuously increase on the way toward B. If, on the contrary, it has, at B, a considerable velocity toward A, it will be retarded by the opposing pressure difference, since it has continuously a somewhat greater pressure in front of it than behind it. It is again so that the velocity at B, where the pressure is smaller, is greater than at A. The mathematical computation for a steady flow in an ideal fluid leads to the conclusion that the sum of the pressure p and the quantity $\rho \frac{v^2}{2}$ (in which ρ = density and v = velocity) is constant on the same streamline. This relation, established by Daniel Bernoulli in 1738, and often referred to as "Bernoulli's theorem," is closely connected with the energy theorem of theoretical mechanics. If a ball is allowed to roll down a smooth surface, shaped as shown in Fig. 8, its velocity will be the greatest at the lowest point and the least at the highest point and $h + \frac{v^2}{2g}$ will be a constant, the height h here playing the same part as the pressure does in the flow. In steady potential flows, moreover, $p + \rho \frac{v^2}{2}$ is constant not only along any given streamline, but throughout the whole field of flow.

We will now apply this principle to the flows of Figs. 3, 6 and 7. At the point A of these flows the fluid comes, for an instant, entirely to rest, so that, according to Bernoulli's theorem, the pressure must here be the greatest, for $\rho \frac{V^2}{2}$ to be greater than in the undisturbed flow (if V is the velocity of the undisturbed flow with reference to the object or, from another viewpoint, the velocity of the object with reference to still air). The pressure is the lowest at the point B, where the condensation of the streamlines shows the maximum velocity. The pressure at the point C is the same as at A. In the symmetrical flow (Fig. 3), the pressure at B' is also the same as at B. In unsymmetrical flows, on the contrary (Figs. 6-7), the pressure at B is considerably smaller than at B' and hence the resultant of the pressures is a force in the direction B'B, i.e., the Magnus effect, for which the term "Quertrieb" (transverse force) was also proposed. No resistance in the direction of motion can be deduced from the pressure distribution in either of the three flows. This is closely connected with the replacement of the actual fluid by the ideal fluid. The overcoming of resistance means the performance of work and could therefore have an equivalent in the ideal fluid only in the kinetic energy remaining in the fluid. If, however, the flow closes behind the object just as it opens in front of the object, there is then no room for such an energy and consequently for any resistance. Rayleigh's calculation and, consequently, our own remarks apply only to a very long cylinder, for which the conditions at the ends are disregarded. They are not applicable to a short cylinder. Even in the ideal fluid, with the circulatory motion at the cylinder ends, kinetic energy develops in the form of eddies which remain in the flow and therefore produce a corresponding resistance (a so-called induced drag, the same as for airplane wings). From this the conclusion (confirmed by experiments) is drawn that the Magnus effect can be observed, in its approximately full development, only in connection with very long cylinders and that it occurs in connection with spheres only in a form that is greatly affected by eddies.* All previous observations were made, however, with relatively short bodies. The Göttingen experiments of 1923 were the first to be performed with sufficiently long cylinders.

Two further points should be noted here. First, the above-described pressure distributions can be intelligibly explained in another way. If we consider a fluid particle which is moving along a curved streamline, we can easily see that, in order to keep the particle in the curved path, a force must be continuously acting on it which tends to deflect it toward the

* In my "Tragflugeltheorie" (wing theory), Part II ("Göttinger Nachrichten," 1919, p. 134), the lift decrease, given by equation 68, of the magnitude $W \tan \delta$ (δ = angle between the direction of the wind and the direction of the eddy axes, which are oblique on account of their own motion; W = induced drag) can be so great for short cylinders that, with an increase in the circulation, the lift may again decrease or even become negative. This phenomenon may be explained by the suction force of the eddies, which acts in the direction of their axes.

concave side of the path curve. This result is accomplished by the pressure on the convex side being somewhat greater than on the concave side. (It may also be said that the particle, in its tendency to go straight ahead, presses on the convex side thus producing a centrifugal force.) If we now follow these pressure differences in directions at right angles to the streamlines till we reach the undisturbed flow where the pressure is equalized, we again find that the pressure must be positive in the vicinity of A, where the streamlines are convex toward the cylinder, and negative in the vicinity of B, where they are concave toward the cylinder. That a quantitative evaluation of this principle will give the same pressures as Bernoulli's theorem, is guaranteed by the connection which these things find in the laws of flow.

The second point concerns the value of the Magnus effect. By computing the pressure distribution, Lord Rayleigh found a formula, according to which this force is proportional to the product of the velocity V of the cylinder with reference to the undisturbed fluid and the velocity U of circulatory flow according to Fig. 4. For a length l of the "infinitely long" cylinder, it is

$$P = \rho V U 2 \pi r l,$$

in which r = radius of cylinder and ρ = density of fluid.

The mistake has often been made of confusing this circulatory velocity with the peripheral velocity of the cylinder.

The relation of these two is neither given at the outset nor is it at all simple. Sometimes it can be found only by experiment. In Fig. 6 it is assumed that $U = V$ and in Fig. 7, that $U = 2V$.

The above Rayleigh formula is, moreover, a special case of Joukowski's formula (1906), $P = \rho\ V\ \Gamma\ l$, which holds good for all cases where a flow generates a lateral force, hence for sails, airplane wings, etc., Γ represents the "circulation," which is obtained by multiplying each portion of the line, along any closed curve embracing the power-generating object, by the velocity component falling in its direction and then adding ("integrating") all these products. In potential flows, this "circulation" has very remarkable properties. In ordinary potential flows (e.g., Fig. 3), it is zero for any closed curve. In circulatory potential flows (e.g., Figs. 4, 6 and 7), it is likewise zero for every closed curve which does not inclose the object flowed around, but it is constant for every closed curve which embraces the object once, so that its value Γ is a measure for the circulatory motion. If r (Fig. 4) is the radius of any given streamline, then the flow velocity u, when the circular streamline is chosen as the closed line, is to be inserted full, since it falls exactly in the direction of the line element and we have $\Gamma = 2\ \pi\ r\ u$, from which, due to the constancy of Γ, it follows that u must be inversely proportional to the distance r.

The recently won knowledge of the circulation principle renders it possible to formulate more accurately the important theorem of Sir William Thompson, namely, that, in a homogeneous non-viscous fluid, the circulation along every line consisting permanently of the same fluid particles, can not change with the lapse of time. The theorem, under the conditions named, is universally applicable, not only to "potential motions," but also to all eddying motions. If we now imagine a cylinder at first without rotation, then (according to the preceding statements) there is no circulation (Fig. 3). If it is subsequently set in motion, it is impossible to see how circulation can suddenly set in contrary to the Thompson theorem. The matter is thus quite hopeless from the standpoint of the ideal fluid, in spite of the very satisfactory streamline pictures and pressure distributions, since it is impossible to see how the circulation can come into existence.

The solution for the simple flow around the non-rotating cylinder, as shown in Fig. 3, is, however, when carefully considered, no more satisfactory, since we know that in a real fluid such a cylinder is far from offering no resistance and we observe, even in actual fluids, forms of flow, quite different from Fig. 3, which are filled with eddies behind the cylinder. We will find that, with the explanation of the deviation of the simple flow, with resistance, from the ideal flow of Fig. 3, we will be in possession of the key for explaining the Magnus effect.

The ground for denying the theory of the ideal fluid, as regards the problems, can be stated. The forces of internal friction are so small in the interior of the almost non-viscous fluids (to which water and air belong) as to be negligible in comparison with the forces of inertia, but in a thin layer immediately on the surface of the immersed body or of the stationary walls they are of the same order of magnitude as the forces of inertia. If we imagine the viscosity of the fluid to be continually decreasing, the specific frictional effects in this layer will not be diminished, but the boundary layer will become thinner.

It is obvious that such a layer must exist, since all experiments on the behavior of viscous fluids demonstrate that the boundary layer clings to the body, i.e., is relatively at rest. The next layers glide over one another, so that the velocity of each successively more distant layer is greater than that of the preceding layer. There is thus created about the body an enveloping zone, in which occurs the transition from zero to the velocity not affected by the viscosity. This transition is brought about by the forces of friction which, as calculated on the basis of the volume element, are of the same order of magnitude as the pressures produced by the inertia of the free fluid, since the velocities in the enveloping zone differ by finite quantities from those in the free current. The nature of the velocity distribution in the enveloping zone is illustrated by Fig. 9. Its thickness δ may be practically assumed to be from 1/300 to 1/50 of the diameter of the cylinder, according to the viscosity.*

The next problem is manifestly connected with the laws of motion of the fluid in the enveloping zone, commonly called "boundary layer." These laws are quite susceptible of computation, which is, however, of a rather difficult nature.

The most important results can also be made intelligible by qualitative considerations. The particles in the boundary layer are subjected, on the one hand, to the accelerating and retarding pressure differences, the same as in the free fluid, and, on the other hand, to the retarding friction with the wall. We will consider the results of this alternating play in connection with a concrete example. For this purpose we choose the beginning of the motion of a cylinder.** The theorems for the ideal fluid apply with sufficient accuracy to the free fluid. Since everything was at rest in the beginning, the circulation is zero for every closed line and must so remain for the lines passing through the same fluid particles. At first, therefore, only the potential flow without circulation is possible, as shown in Fig. 3, whether the cylinder begins to rotate immediately or not. We will assume that the cylinder does not ro-

* The right measure is the Reynolds number $R = \frac{Vd}{\nu}$, with $\nu = \frac{\eta}{\rho}$ = kinetic viscosity (η = viscosity mass and d = cylinder diameter). It is $\frac{\delta}{d} \sim \frac{1}{\sqrt{R}}$.

** Such problems are only solvable by starting from a state for which the velocities of all the particles can be known. The simplest condition is that of complete rest.

tate and study the relations in the frictional zone. If the acceleration is completed during the changed pressure conditions and the cylinder moves uniformly, the pressure is higher at A and C (Fig. 3) and lower at B. The particles in the free flow gain kinetic energy on the way from A to B and lose it again from B to C. The particles in the boundary layer, however, lose, through friction with the wall, a portion of their kinetic energy and no longer have sufficient momentum to penetrate the high-pressure region at C, but come, instead, to a standstill and return to the pressure case existing from C to B. The conditions are the same as for a sphere on the curved track in Fig. 8, which is somewhat retarded by friction. It will turn back below C, which it can not reach, and then oscillate back and forth. In the boundary-layer flow, the situation differs somewhat from that of the rolling ball, in that a forward force is here exerted on the boundary layer by the free fluid. This causes the backward motion to be less extensive than would otherwise be the case.*

The process is as follows. The inner and most retarded layers first change direction and are followed by the next layers. Only the outermost layers of the frictional zone are carried along by the outer flow. Since the boundary layer from B is always newly retarded material which likewise turns back, there

*The forward force can overcome the backward force for bodies which are very long in the direction of flow. Then there is no vortex formation and the resistance is due only to friction, as in the case of a fish, an airship or an ordinary airplane wing.

is formed between B and C an ever thickening accumulation of fluid, set in rotation by the friction, which moves, under the fall in pressure, toward B and then is carried away as "vortices" into the free fluid. Thus a transformation of the flow, starting from the insignificant-appearing processes in the marginal layer, is gradually completed. In the vicinity of B, the boundary layer separates completely from the wall with the continuous formation of new vortices and leaves between it and the wall a region of weak irregular motions.

The process can be illustrated by a few pictures which I took twenty years ago, when I first experimented with these matters with primitive apparatus. Water containing powdered micaceous iron ore was driven through a small trough by a paddle wheel. The flat particles of this mineral assumed different angular positions in different portions of the eddying current, thereby causing differences in the reflection of sunlight. Figs. 10-12 show three different stages of the flow about a cylinder: Fig. 10, after a very short time; Fig. 11, after a little longer time; and Fig. 12, after a still longer time. Fig. 13 shows the permanent condition, which is characterized by an oscillating motion of the train of eddies.

That the processes in the boundary layer are really the cause of the vortex formation, I was able to demonstrate, likewise 20 years ago, by the following experiment. If a slot is made in the cylinder in the region where the back flow first appears, through which slot the water is continually drawn, the retarded water can thus be carried away, before it acquires a backward motion. The effect, which is shown in Figs. 14-15 is, in fact, the elimination of the vortices and also the separation of the flow on the side of the cylinder where the water is sucked out. (The rubber tubes, used for sucking out the water, show in Figs. 14-15.) It is worth noting that the separation of the flow, which is thus prevented as regards the cylinder, is transferred to the wall of the channel.

This separation is not due to the convexity of the wall as in the case of the cylinder, but to the fact that, without the separation, the flow would be greatly retarded (which would be accompanied by a pressure increase). When the other conditions are such that this pressure increase must occur on a straight wall, it first produces a return flow, and then vortices and finally the deflection of the flow from the wall. The beginning of the vortex formation on the channel wall is shown at a in Fig. 14. If a slot were also made in the wall, the separation could here also be prevented or at least greatly retarded, so that the pressure increase and the retardation of the flow could be permanently maintained.*

* Recently experiments with suction on the wall have been resumed in the laboratory under my management. It has been found that by drawing relatively small quantities of fluid through a series of many fine slots, a large variety of flows can be produced, which differ greatly from the ordinary (as, e.g., the deflection of a free air stream 180° by suction in the channel). Obviously this suction process is applicable to all cases where separation is to be avoided: to airplane wings, sails, turbine and propeller blades, ships' hulls and rudders, etc.

I did not try any experiments at that time with rotating cylinders. In the autumn of 1904 I went from Hannover (where I performed the above-mentioned experiments), to Göttingen. Here I had other tasks at first, besides having to procure apparatus similar to what I had in Hannover, so that it was not till 1907 that I resumed water experiments and investigated, among other things, the flow about two oppositely rotating cylinders, standing in contact in a stream of water. Here it was to be expected that, with a sufficient rotation speed of the cylinders, the vortex forming and the separation of the flow would be eliminated, since here the fluid would not be retarded by the friction on the wall, which moved in the direction of the current, but, at most, would be accelerated. The experiment proved the correctness of this assumption. The walls and bottom of the experimental channel were covered with cloths which were made to move with the current, so that separation would here also be prevented. These cloths, which ran over rollers, caused much disturbance, however. Fig. 16 is a picture of such a flow.* In connection with these experiments, a single rotating cylinder was tried once, without however, much importance being attached to this matter. Fig. 17 is a copy of the only picture retained of this experiment. The pictures, made in Hamburg by Prof.

* The small eddies behind the pair of cylinders, due to the boundary layer running faster here than the rest of the stream, had no connection with the vortices producing the separation. The cloudlike disturbances, produced by the walls, running faster than the water on both sides of the lycopodium-strewn current, were of similar origin.

Ahlborn by strewing lycopodium seed on water and employing flash-light illumination, were technically very imperfect. The photographic method has since been greatly improved. Figs. 18-21 are photographs taken by H. Rubach in 1913-14, which very clearly show the separation phenomena on cylinders. It is manifest that potential flow was first present. The incipient return flow shows only in a narrow zone. The pair of eddies then increased rapidly, but gave rise to secondary separation phenomena and eddies, where the pair came in contact with the cylinder. Subsequently, the pair was completely interspersed with such secondary eddies and became continually more irregular and finally disintegrated into an oscillating flow with the continual formation of new eddies.

As regards the explanation of the origin of the circulation flow on the rotating cylinder, which we still needed for a satisfactory explanation of the Magnus effect, it was made very simple by the previously prepared exposition. With a sufficiently high rotation speed, no retardation occurred on the accompanying side and hence no separation, but vortices were formed on the contrary-running side, the same as on the cylinder with the suction. The circulation is always zero for a line (a b c d a in Fig. 22) rounning around both the cylinder and the eddy entirely in the free current. If we add the line b d running through twice in the opposite direction, nothing is thereby changed, since one direction just offsets the other.

From the paths considered, there can be produced, however, two new closed paths, a b d a and c d b c. The last line, which embraces the eddy alone, has a circulation, however, and hence the line a b d a, surrounding the cylinder, must have the same circulation in the opposite direction. The eddy passes along with the current, and the circulation around the cylinder remains.* When the cylinder is rotating slowly, one of the eddies is smaller and the other larger than when the cylinder is not rotating. The circulation around the cylinder is then equal to the difference between the circulations of the eddies.

In order to correct a common error, it should be noted that the kinetic energy of the circulatory flow has no connection with the frictional resistance of the air which the cylinder must overcome when rotating. As my coworker, J. Ackeret, has demonstrated (in a hitherto unpublished research), the cylinder must overcome a resistance during the development of its circulation. The requisite work is the equivalent of the flow energy produced.

The air friction has simply a loosening effect. Its only offset is that when the circulation (through a change in the rotary speed of the cylinder or in the velocity of the wind) no longer corresponds to the normal condition, more eddies are

* The Thompson theorem does not apply in the frictional zone, so that closed lines, which pass anywhere through fluid material coming from the frictional zone, can have a circulation differing from zero.

produced in one rotational direction than in the other, until, by their passing off, a circulation is produced which corresponds to the momentary condition.

We will now more closely investigate the flow of Fig. 7 from the standpoint of our knowledge of the behavior of the boundary layer. If the cylinder has a peripheral velocity greater than the maximum flow velocity, the boundary layer is not retarded, but is everywhere accelerated. Hence no further freeing of eddies will occur after reaching the circulation corresponding to this flow. We therefore conclude first, that in such a case, the flow diagram of Fig. 7 will be approximately attained and, second, that the corresponding transverse force represents the theoretical maximum.* How great must the peripheral velocity of the cylinder be, in order to produce this condition? At first the theory for the maximum flow velocity according to Fig. 3 (at B and B') gives the value 2V. The peripheral velocity U, of the additional flow according to Fig. 4, is therefore also 2V, so that we have the velocities 4V and 0 at B and B'. Hence the above consideration holds good for peripheral velocities u, which are greater than 4V. Nevertheless, it is to be expected that we can go somewhat below the value 4V, because a slight retardation at

*This conclusion is not absolutely obligatory, because fluid particles carried along by the cylinder are thrown off by centrifugal force and may affect the neighboring flow, whereby the circulation can be increased somewhat in excess of the value corresponding to Fig. 7. This effect, however, can not be very great, so that the above statement is approximately correct. It is also corroborated by experiments mentioned farther on.

the point of maximum flow velocity can manifestly do no harm, since it will be offset by the impulsion at the points of minimum flow velocity.

The maximum theoretical power can now be given in accordance with our previous calculations. $\Gamma = 2 \pi r U = 4 \pi r V$, and hence $P_{max} = 4 \pi \rho V^2 r l$. In order to obtain the coefficient of lift c_a,* we divide P by the frontal area of the cylinder $F = 2 r l$ and by the dynamic pressure $q = \rho \frac{V^2}{2}$ and obtain

$$c_{a\ max} = \frac{P}{Fq} = 4 \pi = 12.57 \text{ **}$$

This coefficient of lift is about ten times as large as the values ordinarily obtained for airplane wings. This is due to the fact that the flow is deflected by the rotating cylinder very much more than it would be by an airplane wing. If we study the pressure distribution of the flow in Fig. 7, Bernoulli's theorem gives us the answer that, instead of $A = C$, there is a positive pressure equal to the simple dynamic pressure $\rho \frac{V^2}{2}$ opposite the undisturbed current. Instead of B, on the contrary, where the velocity $v = 4 V$, the dynamic pressure is 16 times as small as at A and the negative pressure opposite the pressure of undisturbed flow is therefore

*This term is borrowed from aviation, in which the force corresponding to the transverse drive of a sail is called "lift."
**Through no fault of mine, it was announced in the daily newspapers that I had this information in 1904. This is not so. I do not remember the exact date, but I think I first knew it in 1918.

15 times the dynamic pressure. The far greater portion of the transverse force is therefore due to the effect of suction. This is also very obvious from the study of Fig. 7, if we bear in mind the centrifugal effects in the fluid, which are manifestly very important in the portion above the cylinder.*

Experiments with rotating cylinders, spheres and other bodies have long stood on the list of research tasks for the aerodynamic laboratory under my charge. They were brought to our attention anew by the above-mentioned theoretical considerations, which made an investigation deem desirable. This had to be postponed, however, on account of more urgent tasks. The decisive impulse was finally furnished by the circumstance that in 1923 we came into possession of very rapid small electric motors, which my faithful coworker of many years, Dr. Betz, had developed for driving the propellers of airplane models (cf. Ackeret, "Zeitschrift für Flugtechnik und Motorluftschiffahrt" 1924, p. 101). This caused Ackeret, who was greatly interested in everything pertaining to boundary layers, to investigate the rotating cylinder. In order to approximate, as closely as possible, the conditions assumed in the theory, the cylinder was placed between two parallel walls, so that the process of flow would be the same in all planes parallel to both walls and all

* Also for ordinary cambered airplane wings, the suction on top is greater than the pressure on the bottom, although not to so great a degree as for a rotating cylinder.

detrimental currents around the ends of the cylinder would be avoided. The peripheral velocity was increased to four times the wind velocity, but at first the highest attainable c_a remained at about 4, instead of its theoretical value of 12.57. An investigation of the flow soon showed us that only the central portion of the cylinder worked correctly, while the current did not cling to it near the ends and was therefore only slightly deflected. I ascribe this deviation from the anticipated flow to a separation of the air stream on both walls corresponding to the separation from the wall as shown in Fig. 15. In order to avoid this, I suggested the addition of circular disks to the ends of the cylinder, to rotate with the cylinder and to prevent the retardation of the boundary layer at the critical points (Fig. 23). The anticipated effect was produced. The flow clung to the cylinder clear to the walls and the coefficient of ~~drag~~ lift c_a increased to about 10, with an approximate 90° deflection of the 20 cm (7.87 in.)-high air stream by a cylinder of 4 cm (1.57 in.) diameter. We could be well satisfied with the result since, due to various deviations produced by friction, we could not expect to obtain the coefficient 12.57.

We had already considered, with reference to the theoretical results, the application possibilities of the rotating cylinder, but saw no practical advantage in connection with any of the things considered (airplane wings, propellers,* wind-

* A propeller model with rotating cylinders was tested by Professor Gümbel in Berlin in 1918.

mills, turbine blades, etc.). I have not changed my opinion. We can greatly reduce the depth in the direction of flow, as compared with wing-shaped structures, by the use of rotating cylinders, since the latter absorb 8-10 times as much force per unit area of the projection, but this generally constitutes no great advantage, because some form without moving parts is structurally more convenient and good wing shapes offer less resistance to the air in the direction of motion. By the substitution of rotating cylinders in place of wings, sails and blades, no saving can be made in the span of airplanes, the diameter of windmills, etc., because the performances of these machines depend on the volume of air utilized per second, which is chiefly determined by the span or diameter.

Unfortunately, we did not at that time consider the case of a ship's sail where the conditions are different and afford many advantages for the rotating cylinder. This discovery was reserved for Mr. Anton Flettner, the inventor of the Flettner ship rudder. He had previously had various experiments for determining the properties of his rudder* performed in the lab-

*This rudder differs from the ordinary steering rudder of a ship, in that it is free to move on its axis like a wind vane and is not turned forcibly by a rudder engine, but is operated by a small auxiliary rudder on its trailing edge, whereby the auxiliary rudder plays the same role with respect to the main rudder as the ordinary rudder does with respect to the ship. In this way the force required for steering is very much reduced even for very large ships. Moreover, the steering action is much steadier, even in a heavy sea, since the rudder can follow the changing currents. In this connection, compare also "Naturwissenschaften" 1924, p. 1106 (E. Forster, "Die neue Entwicklung des Schiffsantriebs").

oratory under my charge and then undertook to apply the principle of his rudder to the sail of a ship. Sailing vessels had gradually come into a very disadvantageous position in comparison with steamships and Diesel-engine ships, due principally to the necessity of a large crew for handling the sails and making the frequent repairs in the rigging. Flettner wished to introduce metal sails, made like the wings of metal airplanes, which would automatically assume the correct position with reference to the wind by means of wind vanes and auxiliary rudders. The storm problem was very difficult, however. The metal sails could not be reefed, though they could always be brought, by means of an auxiliary rudder, into the exact direction of the wind, so that they received no lateral pressure. What if, however, the auxiliary rudder should be damaged in a storm and remain in such a position that the sails would receive the full pressure? There was the further disallusionment that the old type of sail, when correctly adjusted to the wind, was not so bad as we had been inclined to assume, but generated forces equal to about 80% of those produced by metal sails of the same size. The metal sails had to be very large, therefore, in order to fully replace the old sails.

Mr. Flettner therefore began to search for some other substitute. When informed of the Göttingen experiments with the rotating cylinder, he quickly decided to have the availability of such cylinders for his sail ship investigated and made arrangements with us for this purpose. Our previous experiments enabled us to suggest immediately, as the most favorable, the very form which was subsequently installed on his ship. For the reasons already mentioned, each cylinder had to be long, and projecting disks were added to both ends. The free upper disk had somewhat different functions than the previously described disks next to the walls (Fig. 23). Without them, air would have been drawn from the rear side of the cylinder into the negative-pressure region and would have dissipated the circulatory flow throughout a considerable portion of the length of the cylinder and indeed all the more, the greater the negative pressure would have otherwise been. Of course the disk had to rotate also, so that the already mentioned separation would not take place. The disks afforded the further advantage, which was clearly demonstrated in the experiments, of decreasing the induced drag, by dividing the marginal eddy into two eddies flowing away from the walls of the disks, thus producing an effect similar to the transition from a monoplane to a biplane.*

First a cylinder with a built-in electric motor was tested (Fig. 24), once without disks and then with two pairs of disks of different diameters. Fig. 25 shows the combined lift coefficients c_a and drag coefficients c_w (resistance in the direction of the wind divided by Fq) in the form of polar curves, the dashed line (in the lower left corner) being the polar curve of an airplane wing. In Fig. 26, c_a is plotted against u/V (ratio of the peripheral velocity of the cylinder to the velocity of the wind). It is obvious that the cyl-

*Mr. Flettner has established his claim that he learned of this action of the disks from other sources and that he would have used disks on the cylinder even without our suggestion.

inder with disks in the region of $u/V = 4$ attain the greatest lift of c_a = about 10. The cylinder without disks produced a lift of about 4.

Furthermore, the wind forces were determined on a model of the rotor-ship "Buckau" and on a model of an equivalently rigged sailing vessel of the earlier type. The results are plotted in Fig. 27 for a wind (relative to the moving ship) of constant direction and strength, in such a manner that the useful component of the air force falling in the direction of the motion of the ship is plotted in the existing course to the relative wind. The sail areas of the two models (Fig. 28) were in the ratio of 1 : 9.8. On the sailing-ship model the sails had to be reset for every change in the course. The measuring points were farther outward or inward, according to whether the adjustment of the sails was better or worse. The region covered by the measuring points is hatched in Fig. 27. Adjustment is possible on the "rotor ship" only in so far as the peripheral velocity is adapted to the wind velocity. A very great advantage of the rotor is that it does not have to be adjusted for changes in the direction of the wind. On a sailing ship any considerable change in either the direction or the velocity of the wind necessitates a change in the set of the sails. Since this is very laborious, especially on large ships, it is often neglected as long as possible, with a corresponding loss in speed. On a rotor ship, the correct adjustment is automatically made and the revolution speed can be very easily corrected by turning a hand wheel, which controls the electric motor which drives the rotor. Only when the wind changes from starboard to port is it necessary to reverse the rotational direction of the rotors. By rotating the two rotors in opposite directions, the ship can be made to turn in its place. When the wind increases, u/V and hence c_a automatically decrease, so that the wind force increases more slowly than on a sailing ship, where it is necessary to reef the sails. A further weakening is possible by reducing the rotational speed. If, in a heavy storm, the power is entirely switched off from the rotor, the effect of the wind is then very small, so that $c_a = 0$ and $c_w = 0.3$.* The drag is then less than that of the empty rigging of an equivalent sailing vessel.

Fig. 29 represents the experimental ship "Buckau," of 600 metric tons, before and after conversion from a sail ship to a rotor ship. Fig. 30 is a view from the captain's bridge toward the front rotor, whose details can be readily recognized. The loading mast in the middle of the ship serves for lifting the freight.

At a trial trip of the "Buckau" on November 12, 1924, I was able to satisfy myself regarding the fine construction of

* The greater value in Fig. 25 is due to the fact that, with the stationary model cylinder, the critical speed (accurate critical Reynolds number) had not been reached. (Cf. Wieselsberger, "Phys. Zeitschrift," 1921, p. 321; L. Prandtl, "Festschrift der Kaiser Wilhelm Gesellschaft," Berlin, 1921, p. 178; "Ergebnisse der Aerodynamischen Versuchsanstalt," Munich, 1923, Part II, p. 23.)

the rotors and their driving mechanism by the "Germaniawerft" at Kiel. On the inside of each rotor there is a climbable hollow steel column, rigidly connected with the ship and carrying at its top one of the main bearings on which the rotor turns. The rotor also has a bearing at the bottom. The rotor is driven, through a pair of cog wheels just above the lower main bearing, by an electric motor controlled by the Leonard system.* The rotors are constructed from 1 mm (0.04 in.) sheet iron with an internal stiffening frame and are practically noiseless. According to the testimony of the crew, the maneuverability of the ship was excellent. It has not yet been tested in a storm, because there has been no storm since the completion of the ship. There is, however, no occasion for fear, because the wind forces are very small when the rotors are not running.**

The most important point yet to be settled is whether the rotor ship will be able to compete economically with the steam-

*In the Leonard system a continuous-current motor is driven by a specially adapted dynamo in such manner that the magnetic field of the motor is excited by an external source with constant current strength and that the magnetic field of the dynamo is excited by a regulatable current strength, while both armatures are directly connected with each other. Thus the dynamo supplies a regulatable current of any desired voltage and the motor revolves at a corresponding speed. In addition to the two electric motors, there are accordingly on the Buckau, in order to operate the two towers independently of each other, three small dynamos, one for each tower and one for exciting the magnet fields and for other uses.

**The storm test has since taken place with very good results and the ship has begun its mercantile trips.

ship and with ships driven by internal combustion engines. This seems to have been proved by the calculations which, however, lie entirely outside my field of work. The real proof can naturally be had only by actual tests, which will be made with rotor ships and which will demonstrate many things (cost of repairs, etc.) which can not be anticipated. In my opinion the prospects are good in this respect and it is therefore encouraging to hear that a number of larger engine-driven boats are to be converted into rotor ships which will serve as examples. It is unfortunate that a bit of poetry will thereby be taken again from the craft. It must, however, be confessed that the sailing ship could not be saved anyway. May success crown the new ship which is taking its place!

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Translation by Dwight M. Miner,
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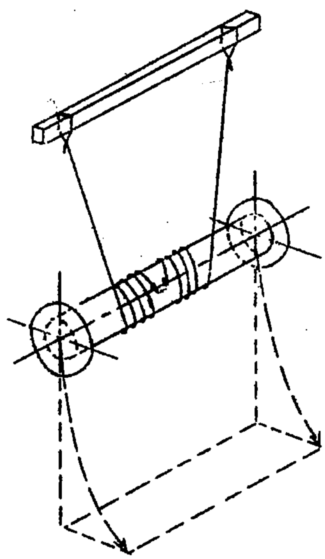

Fig.1

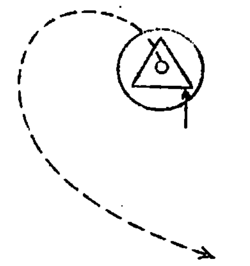

Fig.2

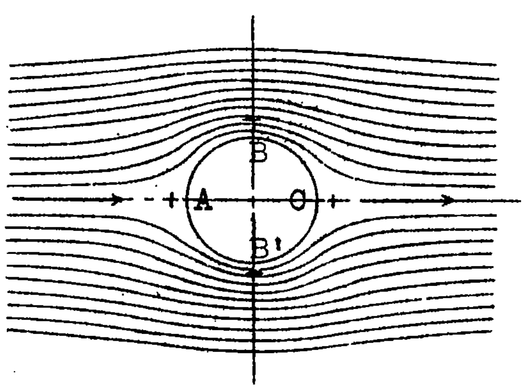

Fig.3 Potential flow around a cylinder.

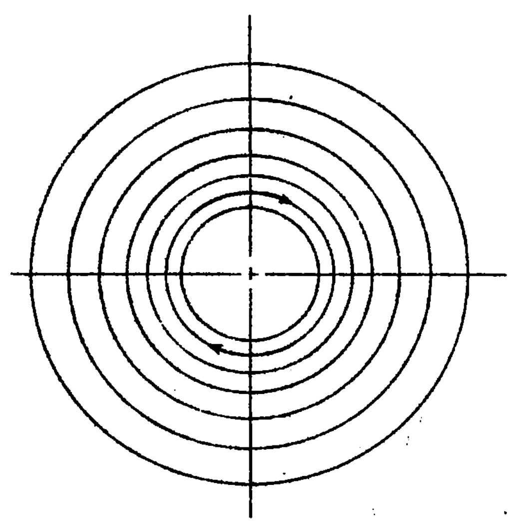

Fig.4 Circulatory flow around a cylinder.

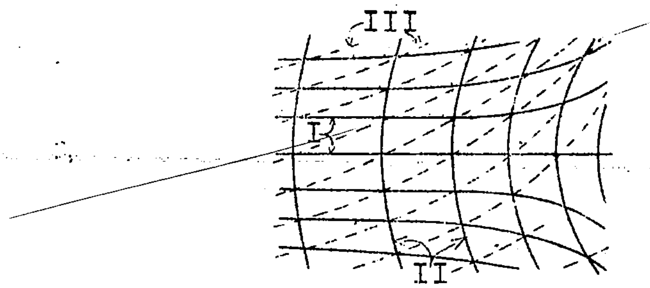

Fig.5 Superposition of 2 flows. The streamline system III is derived from system I and II by drawing the diagonal curves.

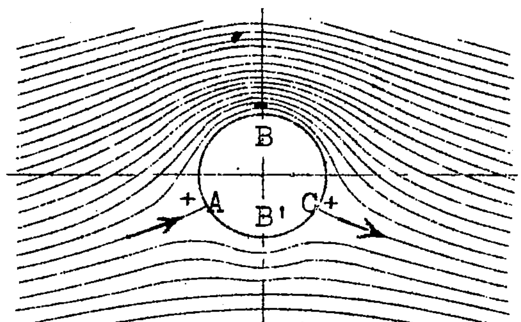

Fig.6

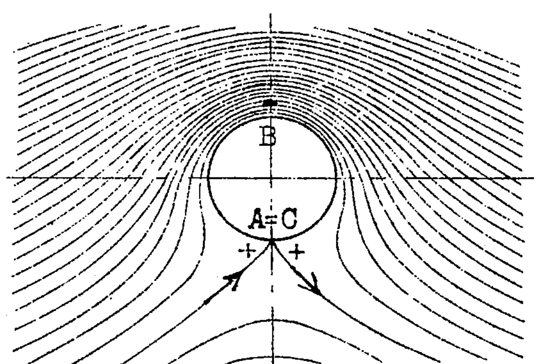

Fig.7
Circulation flows obtained by superposition of figs.3 & 4.

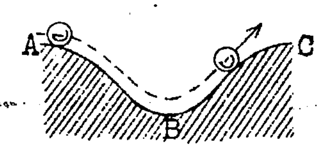

Fig.8 Ball under the influence of gravity.

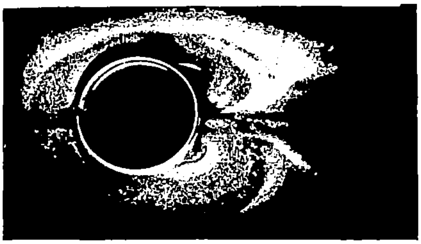

Fig. 10.

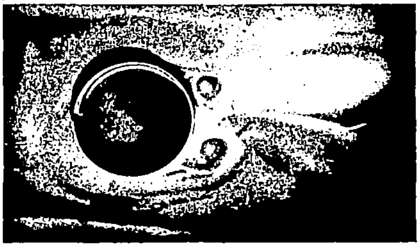

Fig. 11.

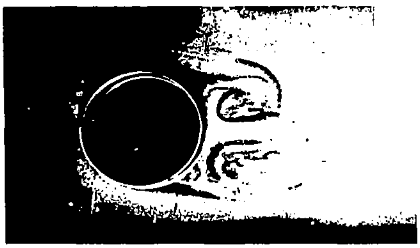

Fig. 12.

Figs.10-13 Flow around a cylinder in various phases of development.

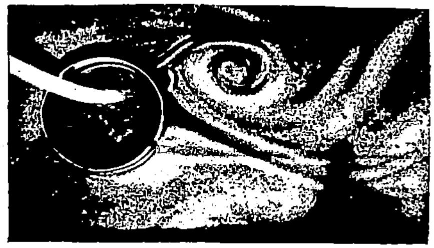

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Fig. 14.

Figs.14-15 Flow around the exhausted cylinder.

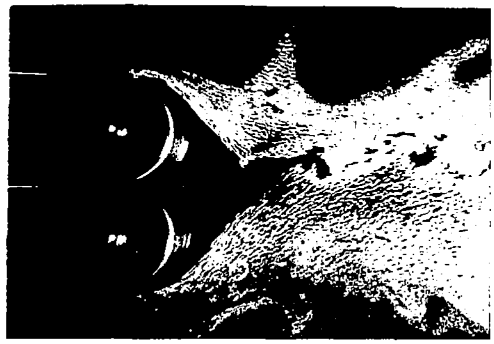

Fig.16 2 oppositely rotating cylinders.

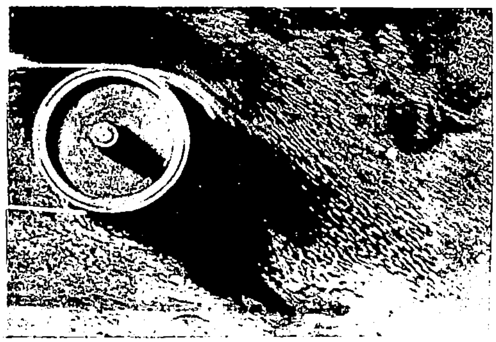

Fig.17 A single rotating cylinder.

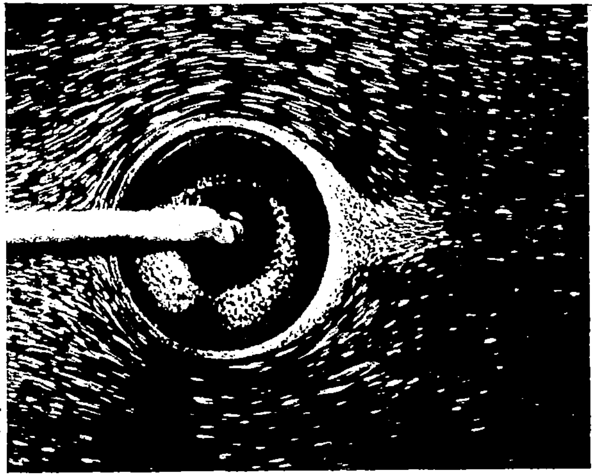

Fig. 18.

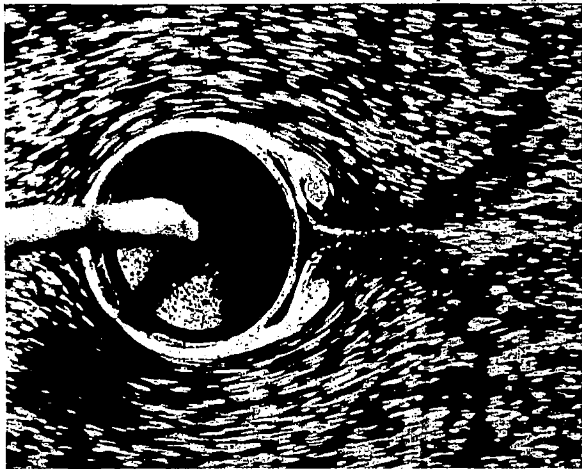

Fig. 19.

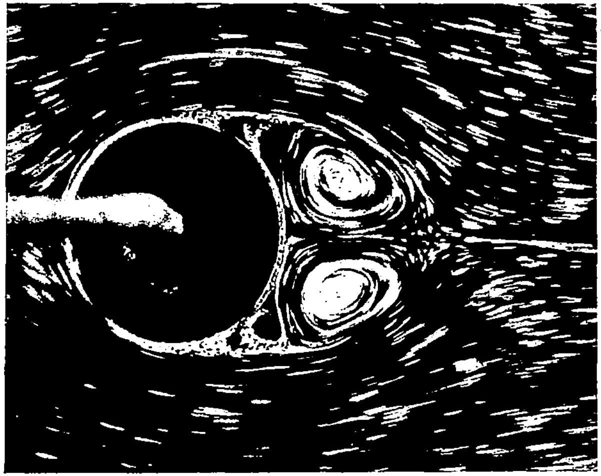

Fig. 20.

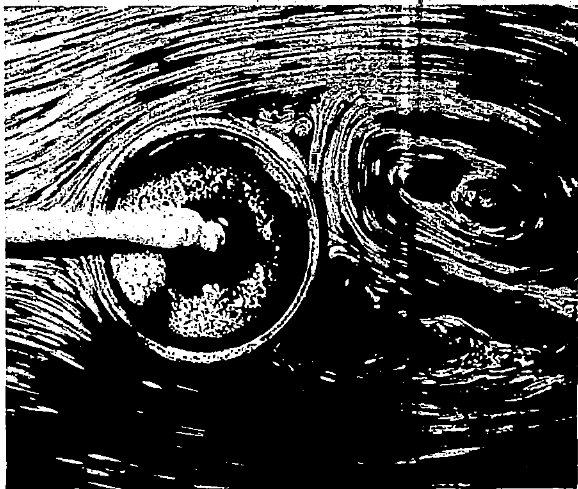

Fig. 21.

Figs.18-21 Formation of vortices and eddies. Photographs by H. Rubach.

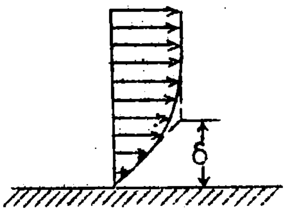

Fig.9 Velocity distribution near the wall.

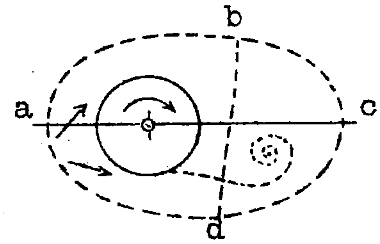

Fig.22 Production of the circulation.

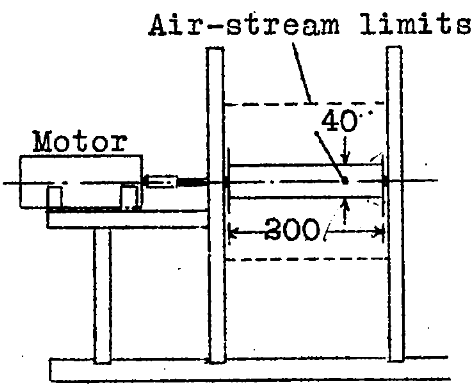

Fig.23 Experimental apparatus.

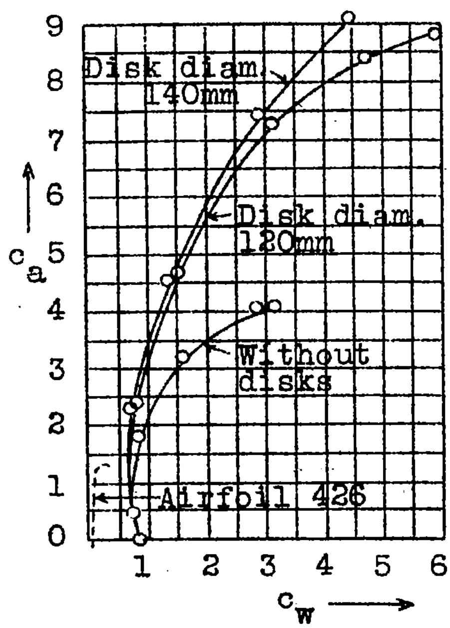

Fig.25 Polar curves. c_a=coef.of lift. c_w=coef.of drag.

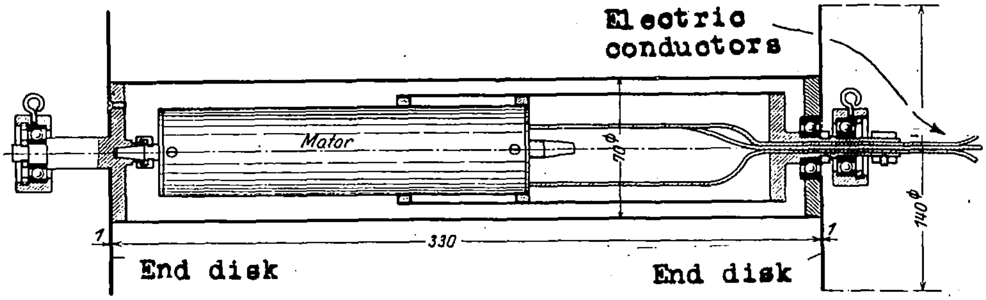

Fig.24 Rotary cylinder with built-in motor.

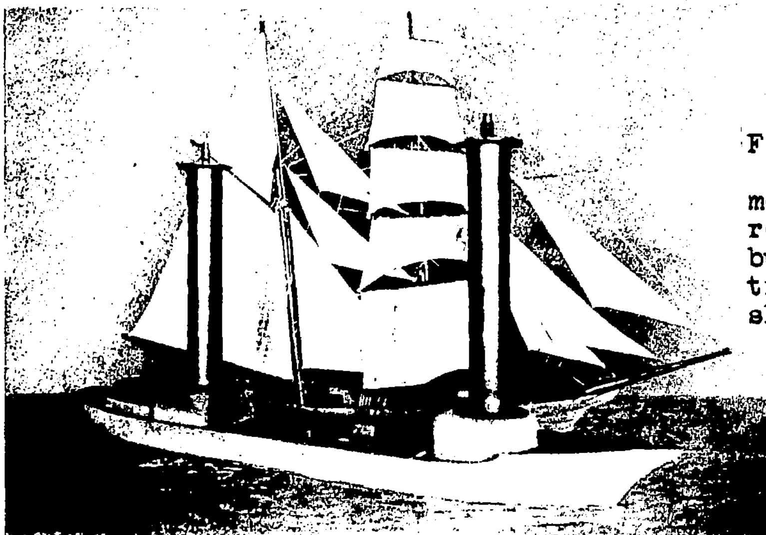

Fig.28 The two ship models. Each rotor has a built-in electric motor as shown in Fig.24

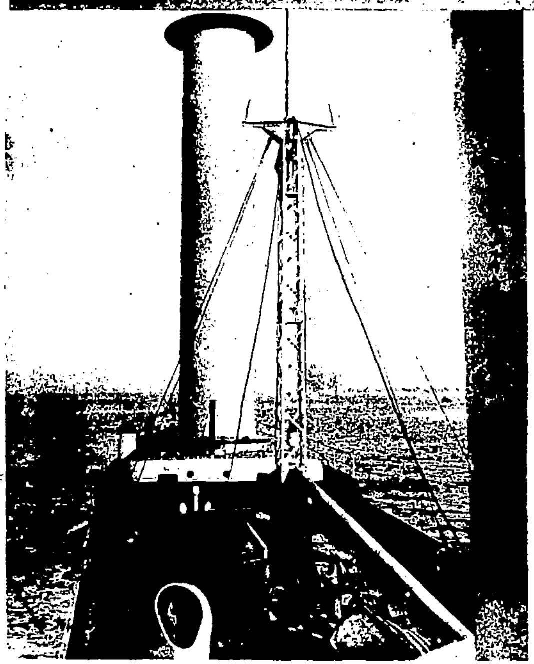

Fig.30 From the bridge of the "Buckau" toward the front rotor.

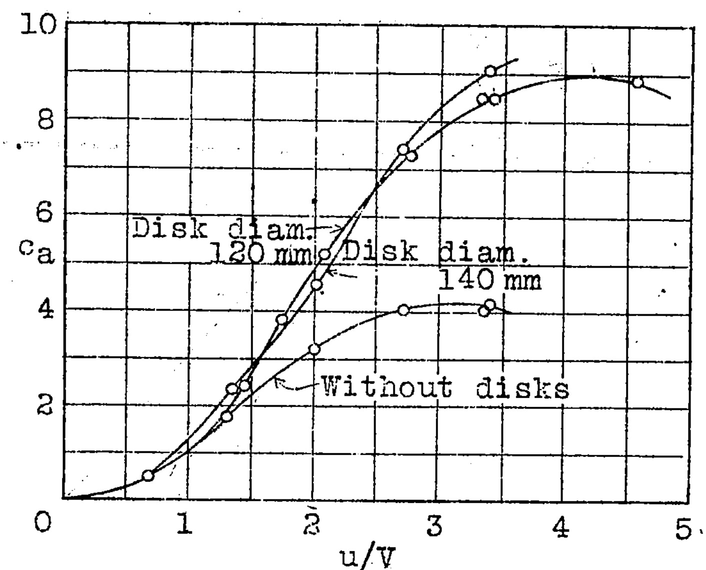

Fig.26 c_a plotted against u/V.

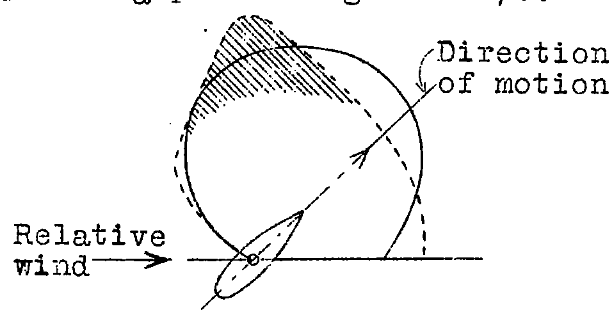

Fig.27 Wind forces on ship models.
———— Rotor ship.
- - - - - Sail ship.

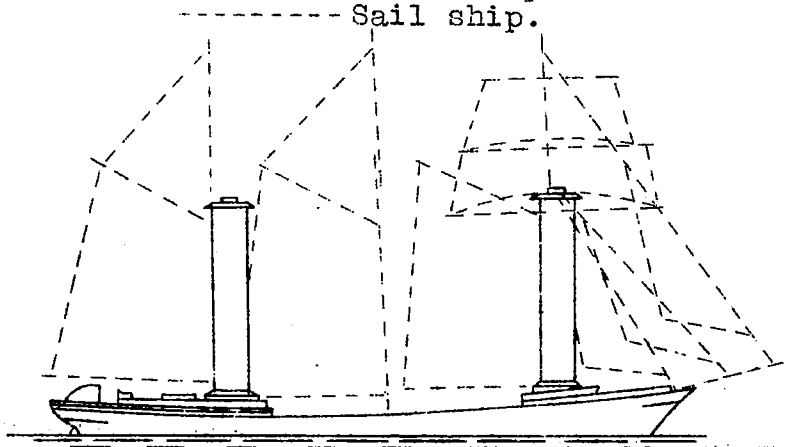

Fig.29 Sail area of "Buckau" before and after conversion.